Integrating Conceptual Foundations in Mathematics through the Application of Principles of Perceptual Learning

Slice & Clone 1 and 2 PLMs
Module Descriptions and Evaluation Results
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This short report is one in a series of reports from a research and development project funded by the Institute of Education Sciences, U. S. Department of Education. This project is conducted collaboratively by research teams at UCLA and at the University of Pennsylvania who have been working together on testing the application of principles of perceptual learning to mathematics education for the past 9 years. Our team includes researchers in perception, cognition, and development; a professional curriculum developer; an educational software developer and programmers; and experienced K-12 educators. We also have strong working relationships with mathematicians and mathematics educators. Through the use of web-based learning technology, we are tapping students’ natural capacities to engage in perceptual learning—a kind of learning that characterizes expert performance in many areas but which has generally been neglected in K-12 classrooms. Perceptual learning is especially effective in leading learners to quickly and easily recognize important structural patterns and relationships across varying contexts. In this project, we apply well-established principles of perceptual learning to improve students’ learning of challenging mathematical concepts related to measurement and fractions. The outcome of this project is a series of curriculum modules consisting of fully evaluated web-based Perceptual Learning Modules (PLMs) and accompanying classroom lessons which are designed to supplement significant portions of the mathematics curriculum for Grades 5-8, as outlined in current NCTM standards, Curriculum Focal Points, and commonly used mathematics curricula.

The results from these studies show dramatic and long-lasting learning gains for middle school participants in the intervention groups. Indeed, students in younger grades who use the PLMs are regularly outperforming students who are a year or more advanced in their performance on the math assessments used in the studies. This report describes two modules focused on fraction concepts, with an emphasis on partitioning continuous quantities into iterable units and multiplying those units to create or compare new quantities. The report includes results from efficacy studies for each module.
Understanding Fractions: 
The Slice & Clone 1 and 2 Perceptual Learning Modules (PLMs)

Learning Challenges: Fractions
A fundamental learning challenge in elementary and middle school mathematics is mastery of concepts related to fractions and operations with fractions. Indeed, the National Mathematics Advisory Panel’s recently released report\(^1\) singled out mastery of fractions as the most critical need in improving mathematics education in the U.S. to provide critical foundations for Algebra. In this study, we describe a pair of learning modules, known as “Slice & Clone 1 and 2,” which are designed to help students develop a powerful, flexible understanding of fractions and how fractions relate to the operations of multiplication and division.

Perceptual Learning in the Classroom: The Slice & Clone PLMs
The evaluation study involved a two-phase classroom intervention with 41 students starting in the spring of their 6\(^{th}\) grade year and continuing into the fall of their 7\(^{th}\) grade year. The students participated in brief introductory lessons and then worked individually with a sequence of two specially designed learning modules. The software modules, which we call the “Slice & Clone 1” and “Slice & Clone 2” PLMs, present students with many short, interactive, animated learning trials. Learners get immediate feedback on each trial and are also given periodic feedback on their progress through the module. In contrast to many other math learning activities, which emphasize declarative knowledge or procedural calculations, this learning software concentrates the students’ attention and effort on learning to pick up relevant structures and relationships. Trials tend to go quickly, since students are not asked to engage in lengthy calculations or extended problem solving. The items in the learning set are designed so that each student will see many varied examples, to enable them to recognize important structures and relationships quickly and easily in different contexts. In this way, perceptual learning modules accelerate the students’ expertise until they are able to “just see” what is important and relevant in each problem. The software tracks each student’s learning progress by recording their accuracy as well as speed in solving short interactive problems of various types. As the student demonstrates his or her fluent mastery of different subtypes of items, those items are retired from the learning set, allowing the student to concentrate time and effort on areas in need of more practice.

Slice & Clone 1 PLM:
Slice and Clone 1 develops the relationships between partitioning quantities into units and iterating units to create new quantities. It also emphasizes repartitioning and chunking units in flexible ways. The software gives the learner two onscreen tools—a slicer, which can be used to divide a given quantity into units of a desired size, and a cloner, which can take the unit created by the slicer and output a desired number of iterations of that unit to create new quantities. On each trial, the learner uses the slicing and cloning tools to solve a problem of creating a new quantity from a given quantity. For example, if the starting quantity is 4/5 units and the learner is supposed to create a new quantity that is 6/5 units, the learner can use the slicing tool to slice the original length into 4 pieces to create a base unit of 1/5. The 1/5 unit then moves to the cloner, and

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the student can use the cloner to iterate it 6 times to output a new quantity that is 6/5 units. The appendix includes screenshots of a sample trial.

This module anticipates some persistent confusions in students’ ability to simultaneously represent and reason about the quantity that defines the base unit as well as new quantities that are constructed with that unit. Specifically, when the starting quantity is not 1 whole unit, the number of pieces into which that quantity is sliced will not be the same as the denominator of the fraction. For example, in the problem given above, one slices the original quantity into four pieces, not five, in order to obtain a unit that is 1/5 of a whole unit. Similarly, once the student has successfully cloned the 1/5 unit to create the new quantity of 6/5, the 1/5 unit represents 1/6, not 1/5 of the new quantity. The iterable unit must be kept in relation both to the quantity that represents 1 whole (e.g., 5 times 1/5 units) as well as any new quantity that is created by the iterating process, which may be greater than, less than, or equal to 1 whole.

This module is designed to help students come to understand fractions as representing quantities, in which the denominator of the fraction defines an iterable unit and the numerator of the fraction specifies a given number of iterations of that unit. This is a more abstract, powerful and flexible representation of fractions than many students have access to. Many students limit their interpretation of fractions to part-whole relationships that are less than or equal to 1—e.g., interpreting 6/8 in terms of starting with a pizza that was cut into 8 slices of which you “have” 6. Improper fractions are puzzling to students who are limited to this part-whole representation because they do not see how you can have more pieces than comprise a whole quantity.

By using the cloning and slicing tools, students make and unmake various quantities. Items in the learning set vary whether the starting and ending quantities are fractions or integers and whether non-integer quantities that are greater than one are expressed as mixed numbers or improper fractions. The resulting trials vary in levels of difficulty depending on how many mental partitions the child must impose to create a useful slice in the first step of the problem. The software is designed so that the cloner becomes operable any time the student selects a productive slice (whether it is the most economical or not). If the sliced unit cannot be cloned to create the desired quantity, the software provides feedback and a second chance to solve the problem.

**Slice & Clone 2 PLM:**

Slice & Clone 2 focuses on several key ideas related to a more integrated understanding of numerators and denominators in fractions and to equivalent fractions. Partitioning creates a fractional unit and is represented by the denominator of a fraction. Iterating creates a multiple of a unit and is represented by a fraction’s numerator. As the partitioning factor gets larger (i.e., the denominator grows), the number of times these smaller units must be iterated to maintain a given quantity increases by the same factor as the partitioning. Equivalent fractions represent different combinations of iterating and partitioning that result in the same quantity.

Slice & Clone 2 again presents learners with the interactive slicing and cloning tools used in S&C 1. The two modules are similar in that both involve an initial quantity (parent) and a need to use that quantity to create a unit that can be cloned productively to create a new quantity (offspring). However, the two modules differ in focus. In S&C 1, all of the quantities involved are either integers or fractions with the same denominator (including improper fractions and mixed numbers). The focus is on creating new quantities using the same base unit. For example, the parent quantity might be 4/5 and the offspring to be created might be 9/5. On S&C 1 trials, there is always one parent quantity given and one offspring to be created.
In S&C 2, the focus is on equivalent fractions involving different denominators (and not necessarily with the least common denominator). The structure of the trials still involves one parent quantity, but now there are two different offspring quantities to be created. The student has to decide how to slice the parent quantity to get a unit that can be productively used to create either of two offspring that have different denominators. For example, given a parent that is 2 units long, the student has to be prepared to create offspring that are either ¾ or 1/6 units long. If the student successfully slices the parent to yield a unit that can be productively cloned to create either of the desired offspring, then the software responds by using that unit to create one of the offspring and then prompting the student to create the other offspring. (See the appendix for examples of screenshots from S & C 2 trials.)

The representations and tasks embedded in the S&C 2 PLM require the learner to recognize or generate different combinations of iterating and partitioning that result in the same quantity. Implicit in this is the relationship that, as the unit size decreases, the number of iterations must increase by the same factor to maintain an equivalent quantity. Thus if the learner is using units of 1/12 to create a quantity of 3/4, then there must be three 1/12 units for every 1/4 unit. The learner must be able to flexibly partition and rechunk units to maintain the appropriate relationships.

**Study Design**

The S&C 1 and 2 modules were tested in one long study that ran for a year with the same group of students. All students attended the same urban middle school, which serves a racially and ethnically diverse population (55% Hispanic/Latino, 32% African American, 8% Asian, and 4% White), with 81% of the students qualifying for free or reduced lunch.

Students in the experimental group (n = 41) completed the S&C 1 PLM in the spring of their 6th grade year and then did the S&C 2 PLM in the fall of their 7th grade year. There was also a comparison group of uninstructed 7th graders (n = 21) who were tested once in the spring. Students in the PLM intervention condition worked with the learning modules until they met mastery criteria for most or all of the item categories in the learning set. The number of sessions per student thus varied, with students taking an average of 6.5 computer sessions (of approximately 35 minutes each) to complete the S & C 1 module and 8 sessions to complete the S & C 2 module. Here is how the study was laid out:

<table>
<thead>
<tr>
<th>Group</th>
<th>Spring 2008</th>
<th>Spring 2008</th>
<th>Fall 2008</th>
<th>Fall 2008</th>
<th>Spring 2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intervention Group: Started in Spring of 6th Grade and continued into 7th Grade (n = 41)</td>
<td>Pretest on S&amp;C1 and S&amp;C2 subscales (Pretest)</td>
<td>Completed Slice &amp; Clone 1 PLM, followed by an immediate posttest on both subscales (Posttest 1)</td>
<td>Took a delayed posttest on both subscales after a 5 month delay (over the summer, students in this group now in 7th grade) (Posttest 2)</td>
<td>Completed Slice &amp; Clone 2 PLM followed by another immediate posttest on both subscales (Posttest 3)</td>
<td>Took a final delayed posttest on just the S&amp;C 2 subscale (Posttest 4)</td>
</tr>
<tr>
<td>Uninstructed 7th Graders (n = 21)</td>
<td>Tested once on both S&amp;C1 and S&amp;C2 subscales</td>
<td></td>
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The pre/post assessment was made of up two subscales: one group of items tapped concepts related to S & C1 and the other tapped concepts related to S & C2. The assessment included items related to unit partitions,
open-ended fraction division and multiplication problems, comparisons of fraction pairs with varying numerators and denominators (i.e., identify which member of pair is the larger quantity), and open-ended word problems involving fractions. It is important to note that the assessment emphasized transfer and generalization with problems that did not directly resemble the PLM training tasks, in order to assess whether students could successfully apply their understanding of fractions and units outside the PLM learning environment on typical pencil-and-paper math assessment items.

**Study Results: Slice & Clone 1**

On average, the students in the intervention group mastered 9 out of the 11 categories in the S & C 1 PLM, with 73% of the students mastering all 11 categories. As the figure below shows, prior to the PLM intervention, the 6th grade intervention group and the 7th grade control group scored poorly on the section of the assessment that tested concepts related to the content of the S & C 1 module. After students in the intervention group completed the S & C 1 module, their scores improved dramatically. A delayed posttest, which was given after a five-month period during which there was no further intervention (and which included summer vacation), showed that students fully maintained their learning gains, with no drop in performance at all. The learning gains are statistically significant, with large effect sizes (Delayed Posttest vs. Pretest effect size = 1.75; Delayed Posttest vs. Control effect size = 2.79).
Study Results: Slice & Clone 2

Students in the PLM intervention group completed the S & C 2 PLM in the fall of their 7th grade year, just after completing the delayed posttest for the S & C 1 intervention (which served as a second pretest for the S & C 2 intervention). The students took an average of 8 sessions to complete the S & C 2 PLM and met the mastery criteria for an average of 7 out of 8 of the item categories in the learning set. Seventy percent of the students mastered all 8 categories. After they finished the S & C 2 PLM, they took an immediate posttest which included items relevant to both the S & C 1 and S & C 2 content. Five months later, with no further intervention, the students completed a final delayed posttest, which included only items from the S & C 2 subscale. The figure below gives the results of students’ performance on the S & C 2 subscale at each test administration.

As can be seen in this figure, the students in the intervention group show a dramatic improvement in their performance on the S & C 2 assessment items after they have completed the S & C 2 PLM. Again, they maintained these learning gains over a delay of more than 5 months, during which no study-related learning activities occurred. The learning gains are highly significant, with an effect size of .84 for the Posttest vs. Pretest scores for the intervention group and an effect size of 1.14 when the intervention group’s posttest scores are compared to the control group.

Summary

The study results indicate that instructional methods based on perceptual learning can help students succeed in areas of the math curriculum known to be problematic. The robust and durable learning gains demonstrated for the intervention group suggest that the two Slice and Clone Perceptual Learning Modules advance students’ structural insight and fluency related to fractions in ways that the normal mathematics curriculum does not achieve. The PLM is an efficient and effective way to bring perceptual learning into classrooms as a
complement to other forms of instruction. The software—which is able to track each student’s individual progress, to customize itself to the student’s rate of learning, and to certify mastery—provides a set of practical tools for accelerating learning and differentiating instruction in classrooms.

Appendix: Screen Shots of PLMs and Sample Assessment Items

Screen Shots for Slice & Clone 1 PLM:
The software environment makes fraction concepts tangible to learners by providing them with interactive on-screen tools that they can manipulate. They use a “slicer” to cut continuous extents into a desired number of pieces, thus creating a base unit. They can then drop the unit that they have created into a “cloner” that will iterate that unit a desired number of times and output the result. The students’ task is to start with a given quantity (the “parent” quantity) and use the slicing and cloning tools to create a new quantity (the “offspring” quantity).

Below is a series of screen shots that illustrate the different steps as a student interacts with the software to solve a problem.

Step 1: The student is presented with a parent quantity and has to decide how to slice it to get a unit that can be used productively to create the offspring quantity.
Step 2: Here the student has decided (correctly) to slice the parent quantity into 4 pieces.

Step 3: The software carries out the slice to create a unit that is $\frac{1}{3}$. 
Step 4: The unit that the student created in the previous step has dropped down into the cloner, and the student is using the cloner to clone that unit two times.

Step 5: The PLM completes the cloning and outputs the result, along with feedback that the student has completed the task correctly. Notice that as the software carries out the slicing and cloning steps, it also represents the actions in a corresponding multiplication or division number sentence in the left-hand column.
Sample Assessment Items for Slice & Clone 1:

This block is $\frac{3}{4}$ units long. How long is the shaded piece? _______ units

Pretest Mean: 36%  
Delayed Posttest Mean: 98%

This block is 2 units long. How long is the shaded piece? ________ units

Pretest Mean: 17%  
Delayed Posttest Mean: 73%

$\frac{1}{3} + 4 =

Pretest Mean: 17%  
Delayed Posttest Mean: 61%

$2 + 6 =

Pretest Mean: 8%  
Delayed Posttest Mean: 63%
**Screen Shots for Slice & Clone 2 PLM:**

This sequence of three screen shots illustrates an initial problem screen for the S & C 2 PLM followed by the feedback a student gets after making an incorrect slice. The student then gets another chance and makes a correct slice that divides the parent quantity into a unit that can be used to make either of the offspring quantities.
The screen shot below illustrates the second part of a trial from the S & C 2 PLM. The student has been presented with a trial in which she has to slice a 1 unit block to create a unit that can be cloned to create two different offspring quantities with different denominators (2/12 and 2/6). This student has correctly decided to slice the parent to create units of 1/12. The software responds by cloning one of the offspring quantities and then asks the student to complete the trial by cloning the other offspring quantity. The student is in the process of cloning 1/12 four times in order to create 2/6.
Sample Assessment Items for Slice & Clone 1:

\[ \frac{1}{3} \div 5 = \]

Pretest Mean: 13%
Delayed Posttest Mean: 79%
(7th Grade Control Group: 0%)

How many \( \frac{1}{12} \) units in \( \frac{5}{6} \) units?  Answer: __________

Pretest Mean: 23%
Delayed Posttest Mean: 71%

Circle the larger item in the pair:

\[ \frac{5}{9} \quad \frac{11}{18} \]

Pretest Mean: 42%
Delayed Posttest Mean: 87%

This block is \( \frac{6}{10} \) units long. Shade the length of \( \frac{1}{5} \) unit.

Pretest Mean: 32%
Delayed Posttest Mean: 74%

You need \( \frac{1}{4} \) cup of sugar to make one cupcake. How many cupcakes can you make from \( \frac{9}{12} \) cups of sugar?
Answer: __________

Pretest Mean: 28%
Delayed Posttest Mean: 74%

Is there ANY fraction between \( \frac{2}{5} \) and \( \frac{3}{5} \)?

Yes  No
Pretest Mean: 43%
Delayed Posttest Mean: 71%

If you answered “Yes,” give an example ______
Pretest Mean: 6%
Delayed Posttest Mean: 26%
For more information: Contact Christine Massey at massey@seas.upenn.edu.

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